pNRQCD with lattice QCD input

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— Heavy Quark Hadrons at J-PARC 2012 —

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- Quarkonium / Positronium
- Nonrelativistic bound states in QFT / EFT
- NRQED / pNRQED
- NRQCD / pNRQCD / Matching coefficients
- Lattice QCD results

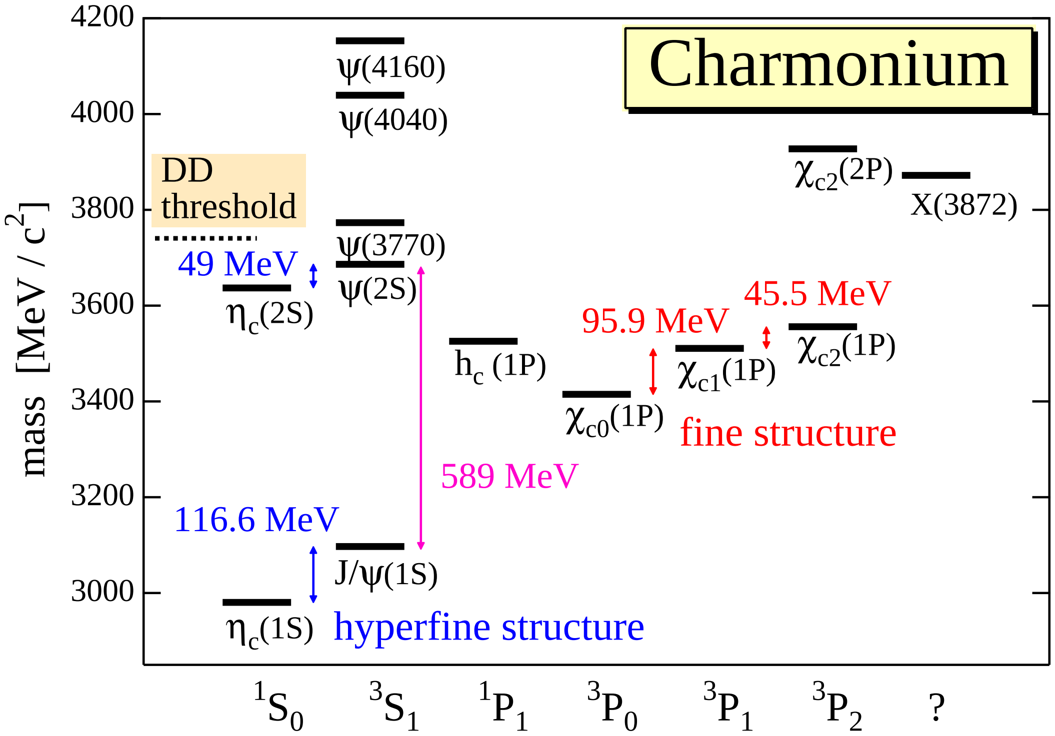
  [Y.Koma, M.Koma & H.Wittig, Phys. Rev. Lett. 97 (2006) 122003]

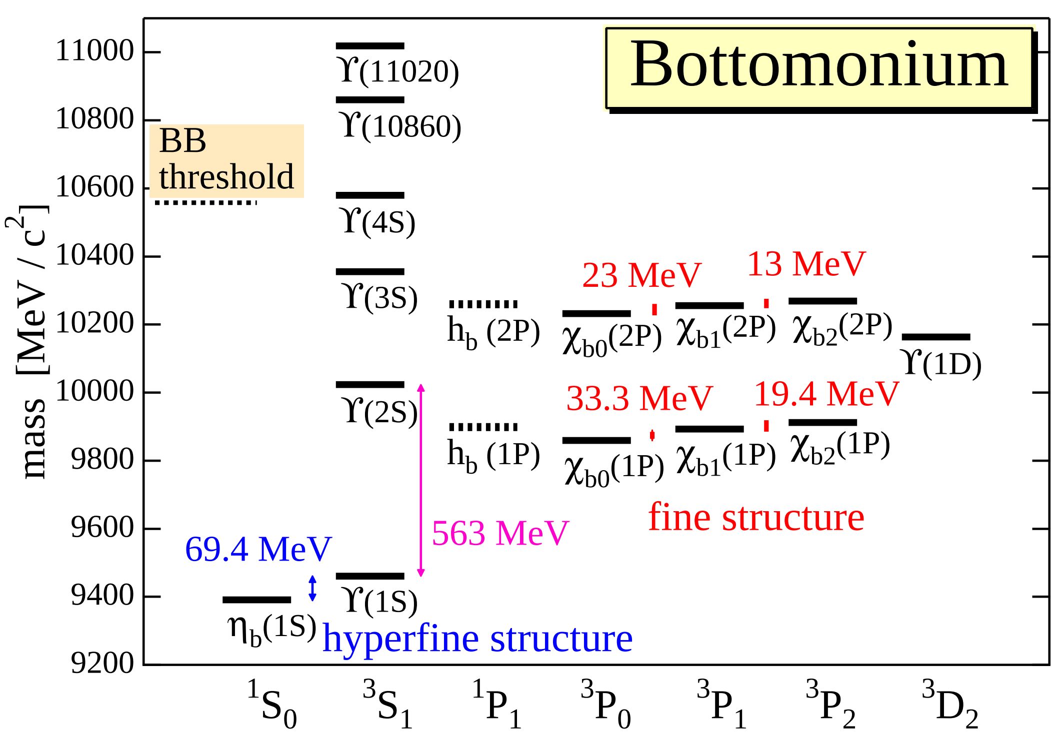
  [Y.Koma, M.Koma, Nucl. Phys. B769 (2007) 79]

  [Y.Koma, M.Koma, Prog. Theor. Phys. Suppl.186 (2010) 205]
- **pNRQCD with lattice QCD input**
- Summary and outlook

Quarkonium

▶ **bound state of heavy quark and anti-quark**

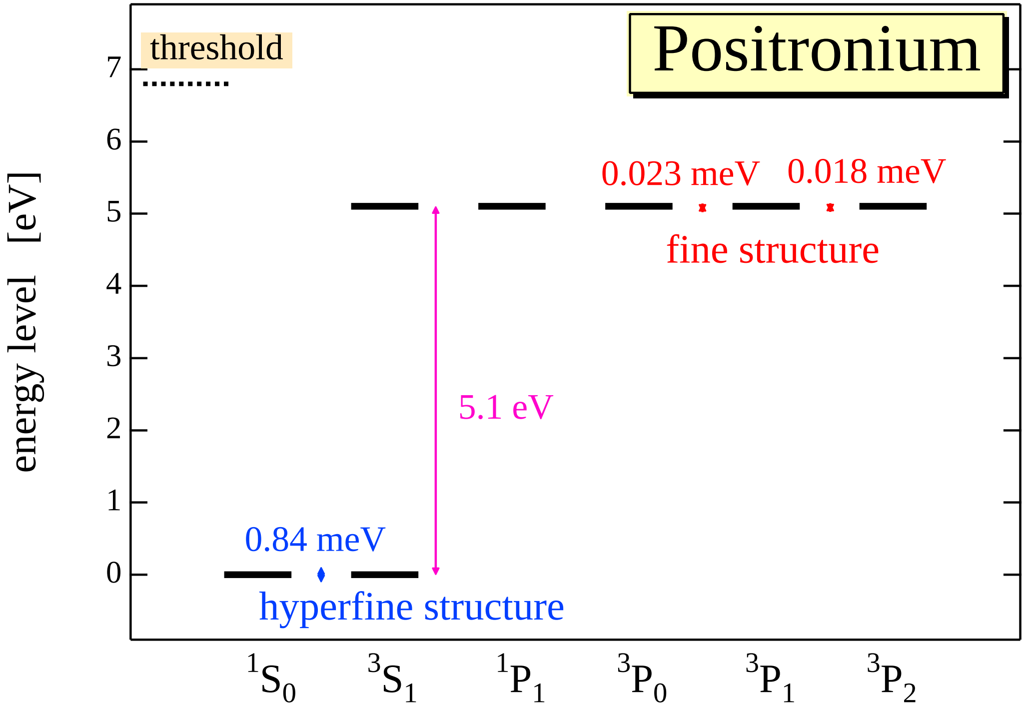




- **precise and rich data, should be explained by QCD**
- **similar to positronium** ⇒ **nonrelativistic system ?**

Positronium

▶ **bound state of electron and positron**



$$\Delta E(^3P_1 - {}^3P_0) = 0.022759(57)\ \mathrm{meV}$$
$$\Delta E(^3P_2 - {}^3P_1) = 0.018200(27)\ \mathrm{meV}$$

$$\Delta E(^3S_1 - {}^1S_0) = 0.841150(3)\ \mathrm{meV}$$

[e.g. Karshenboim ('05)]

- **precise data, should be explained by QED**
- $m = 0.5\,\mathrm{MeV}$, $mv = 4$ keV, $mv^2 = 5$ eV (**nonrelativistic**)

hierarchy of energy scales $m \gg mv \gg mv^2$

Nonrelativistic bound states in QFT

- **how to describe nonrelativistic bound states**
  - **"conventional" perturbative expansion $\Rightarrow$ fails**
  - **"naive" NR reduction ...**

  Dirac (BS) eq. $\longrightarrow$ Foldy-Wouthuysen-Tani trans.
  $\longrightarrow$ Breit-Fermi Hamiltonian $\longrightarrow$ Schrödinger eq.

  leading order $\Rightarrow$ OK, higher oder $\Rightarrow$ fails

  not suit for high precision calculation
  due to missing of high energy contribution

Effective field theories

- hierarchy of energy scales $M_{\mathrm{high}} \gg M_{\mathrm{low}}$
- integrate out the scale above $M_{\mathrm{high}} \Rightarrow$ **EFT**
- in EFT ...
  the scale below $M_{\mathrm{high}} \Rightarrow$ dynamical d.o.f
  the scale above $M_{\mathrm{high}} \Rightarrow$ **effective couplings**
  (matching coefficients)
- systematic improvement ...
  precision is controlled by the small ratio $\dfrac{M_{\mathrm{low}}}{M_{\mathrm{high}}} \ll 1$

NRQED (integrate out $O(m)$)

- **EFT for positronium**

  [Caswell & Lepage ('86), Kinoshita & Nio ('96), Labelle ('97), ...]

$$\mathcal{L} = \psi^\dagger \left(c_F e \frac{\sigma \cdot B}{2m} + c_D e \frac{D \cdot E - E \cdot D}{8m^2} + i c_S e \frac{\sigma \cdot (D \times E - E \times D)}{8m^2} + \cdots \right) \psi$$

$$+ (\psi \to \chi)$$

$$+ \frac{d_s}{m^2} \psi^\dagger \psi \chi^\dagger \chi + \frac{d_v}{m^2} \psi^\dagger \sigma \psi \cdot \chi^\dagger \sigma \chi$$

- **compute scattering amplitude both in QED and NRQED**

  $\Rightarrow$ **matching coeff.** **are fixed to reproduce the same result**

  $\Rightarrow$ **no bound state information**

pNRQED (integrate out $O(mv)$)

- to avoid complication in power counting due to mixture of the scales mv and mv^2 [Pineda & Soto ('98)]
- matching between NRQED and pNRQED is performed by calculating Green functions in both EFTs
- matching coefficients in pNRQED are local in time but nonlocal in space (potential)

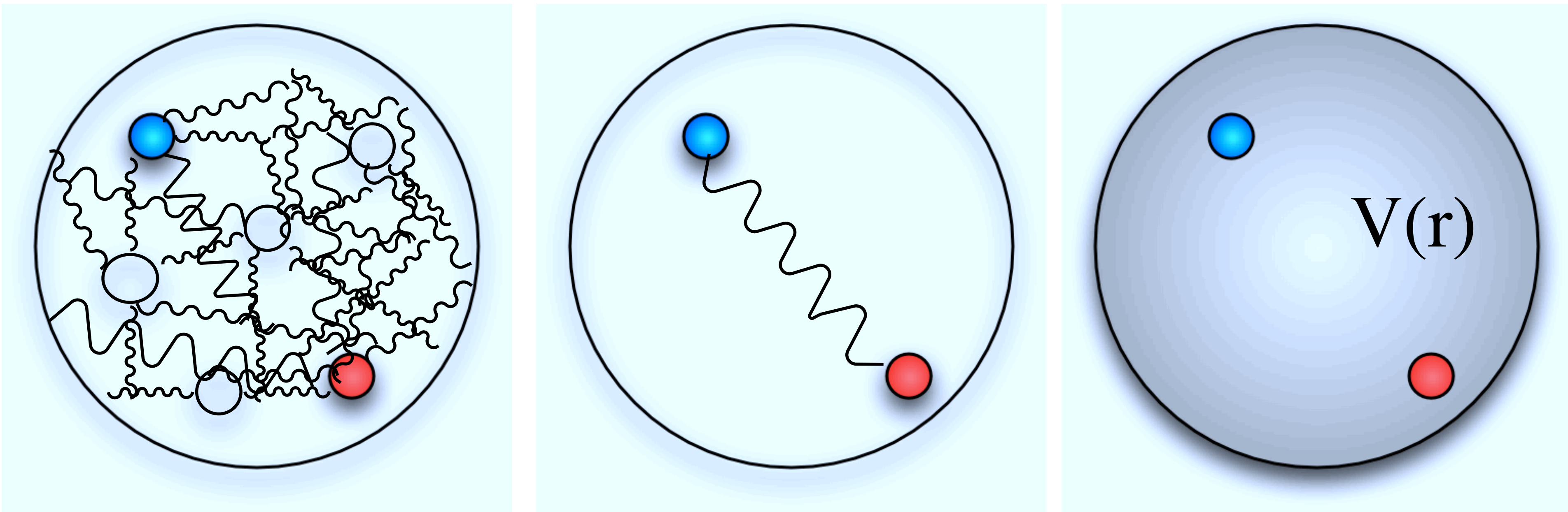
$$V = -\frac{\alpha}{r} + \frac{1}{m^2}\Bigg\{\Big(\pi\alpha(c_D - 2c_F^2) + d_s + 3d_v\Big)\delta(\boldsymbol{r}) - \frac{\alpha}{2r}\left(\boldsymbol{p}^2 + \frac{1}{r^2}\boldsymbol{r}\cdot(\boldsymbol{r}\cdot\boldsymbol{p})\boldsymbol{p}\right)$$

$$+ (c_S + 2c_F)\frac{\alpha}{2r^3}\boldsymbol{l}\cdot\boldsymbol{s} + c_F^2\frac{\alpha}{r^3}s_{12} + \left(\frac{4\pi\alpha}{3}c_F^2 - 2d_v\right)\delta(\boldsymbol{r})\boldsymbol{s}^2\Bigg\}$$

From QED to EFTs

- hierarchy of energy scales $m \gg mv \gg mv^2$
- integrate out the scale m (NRQED) and mv (pNRQED)

QED $\Rightarrow$ NRQED $\Rightarrow$ pNRQED



NRQCD (integrate out $O(m)$)

- **EFT for quarkonium**

  [Caswell & Lepage ('86), Manohar ('97), Pineda & Soto ('98)]

$$\mathcal{L}=\psi^\dagger\left(c_F g\frac{\boldsymbol{\sigma}\cdot\boldsymbol{B}}{2m}+c_D g\frac{\boldsymbol{D}\cdot\boldsymbol{E}-\boldsymbol{E}\cdot\boldsymbol{D}}{8m^2}+ic_S g\frac{\boldsymbol{\sigma}\cdot(\boldsymbol{D}\times\boldsymbol{E}-\boldsymbol{E}\times\boldsymbol{D})}{8m^2}+\cdots\right)\psi$$

$$+(\psi\to\chi)$$

$$+\frac{d_{ss}}{m^2}\psi^\dagger\psi\chi^\dagger\chi+\frac{d_{sv}}{m^2}\psi^\dagger\boldsymbol{\sigma}\psi\cdot\chi^\dagger\boldsymbol{\sigma}\chi$$

$$+\frac{d_{vs}}{m^2}\psi^\dagger T^a\psi\cdot\chi^\dagger T^a\chi+\frac{d_{vv}}{m^2}\psi^\dagger T^a\boldsymbol{\sigma}\psi\cdot\chi^\dagger T^a\boldsymbol{\sigma}\chi+\cdots$$

- **similar to NRQED ...**

  electron $\to$ quark, photon $\to$ gluon, $e\to g$, etc.

Matching coefficients in NRQCD

- **bilinear term** c_s, c_D, c_F [Manohar ('97)]

$$c_F = 1+\frac{\alpha_s}{\pi}\left[\frac{C_F}{2}+\left(\frac{1}{2}-\frac{1}{2}\ln\frac{m}{\mu}\right)C_A\right]+O(\alpha_s^2)$$

$$c_D = 1+\frac{\alpha_s}{\pi}\left[\left(\frac{8}{3}\ln\frac{m}{\mu}\right)C_F+\left(\frac{1}{2}+\frac{2}{3}\ln\frac{m}{\mu}\right)C_A\right]+O(\alpha_s^2)$$

$$c_S = 1+\frac{\alpha_s}{\pi}\left[C_F+\left(1-\ln\frac{m}{\mu}\right)C_A\right]+O(\alpha_s^2)$$

- **contact term** d_{ss}, d_{sv}, d_{vs}, d_{vv} [Pineda & Soto ('98)]

$$d_{ss} = \frac{2}{3}\pi\alpha_s + O(\alpha_s^2)\ , \quad d_{sv} = -\frac{2}{9}\pi\alpha_s + O(\alpha_s^2)$$

$$d_{vs} = -\frac{1}{2}\pi\alpha_s + O(\alpha_s^2)\ , \quad d_{vv} = \frac{1}{6}\pi\alpha_s + O(\alpha_s^2)$$

pNRQCD (integrate out $O(mv)$)

- **general form of the potential in pNRQCD**

$$V(r)=V^{(0)}+\frac{1}{m}(V^{(1,0)}+V^{(0,1)})+\frac{1}{m^2}(V^{(2)}_{\mathrm{SI}}+V^{(2)}_{\mathrm{SD}})$$

$$V^{(2)}_{\mathrm{SI}}=\{p^2,V^{(2)}_{p^2}\}+\frac{1}{2}\{p^2,V^{(1,1)}_{p^2}\}+\frac{2V^{(2)}_{l^2}+V^{(1,1)}_{l^2}}{r^2}l^2+2V^{(2)}_r+V^{(1,1)}_r$$

$$V^{(2)}_{\mathrm{SD}}=(V^{(2)}_{ls}+V^{(1,1)}_{ls})l\cdot s+V^{(1,1)}_{s^2}s_1\cdot s_2+V^{(1,1)}_{s12}s_{12}$$

e.g. $$V^{(1)}=V^{(1,0)}=V^{(0,1)}=-\frac{1}{2}\int dt\, t\langle\langle E(0)\cdot E(0)\rangle\rangle$$

$$V^{(2)}_{ls}=\frac{c_S}{2r}\frac{dV^{(0)}}{dr}+\frac{c_F}{r}\epsilon_{ijk}\hat{r}_i\int dt\, t\langle\langle B^j(0)E^k(0)\rangle\rangle=\frac{c_S}{2r}\frac{dV^{(0)}}{dr}+\frac{c_F}{r}V'_1$$

$$V^{(1,1)}_{ls}=\frac{c_F}{r}\epsilon_{ijk}\hat{r}_i\int dt\, t\langle\langle B^j(0)E^k(r)\rangle\rangle=\frac{c_F}{r}V'_2$$

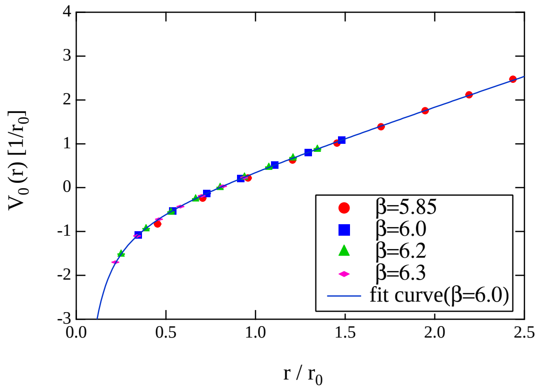
Lattice QCD results

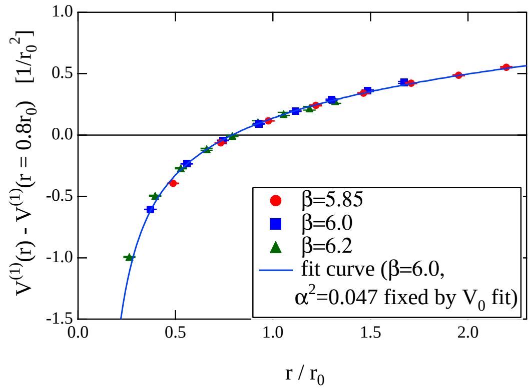
- compute the potential in lattice QCD (quenched)

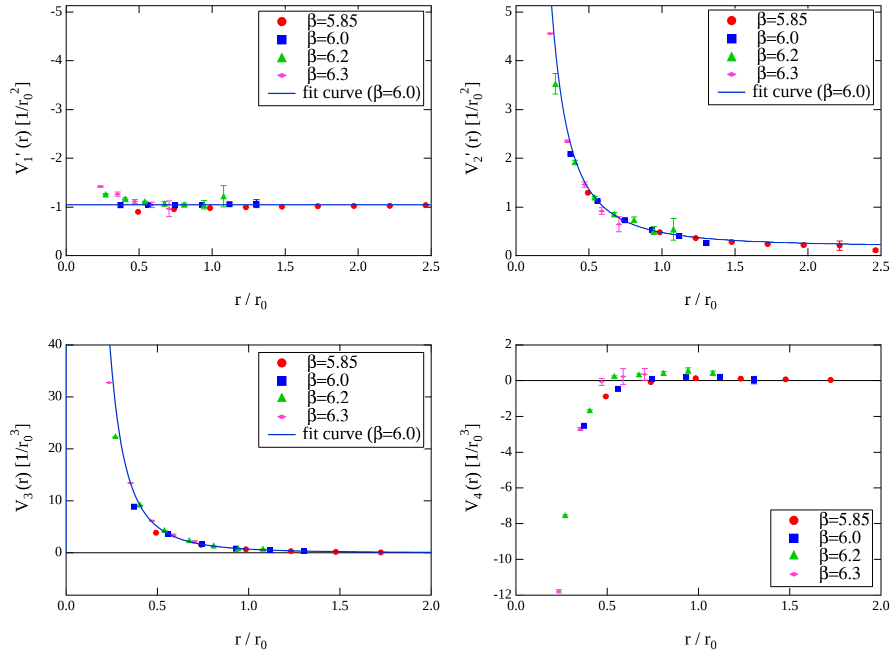
  [Koma, Koma & Wittig ('06ff)]

  - Polyakov loop correlation function, transfer matrix
  - Lüscher-Weisz multilevel algorithm

$$V(r) = V^{(0)} + \frac{1}{m} V^{(1)} + \frac{1}{m^2} V^{(2)}$$







pNRQCD with lattice QCD input

- **solve Schrödinger equation**

$$H^{(0)}\psi = \left(\frac{p^2}{m} + V^{(0)}\right)\psi = E^{(0)}\psi$$

- **compute corrections within the first order perturbation**

$$E = E^{(0)} + \delta E = E^{(0)} + \langle\psi|\delta V|\psi\rangle$$

- **investigate effects of $V^{(1)}$, V_1', and V_2'**

$$\delta E^{(1)} = \frac{2}{m}\langle\psi|V^{(1)}|\psi\rangle$$

$$\delta E_{ls}^{(2)} = \frac{1}{m^2}\langle\psi|\left(\frac{c_s}{2r}\frac{dV^{(0)}}{dr} + \frac{c_F}{r}(V_1' + V_2')\right) l\cdot s|\psi\rangle$$

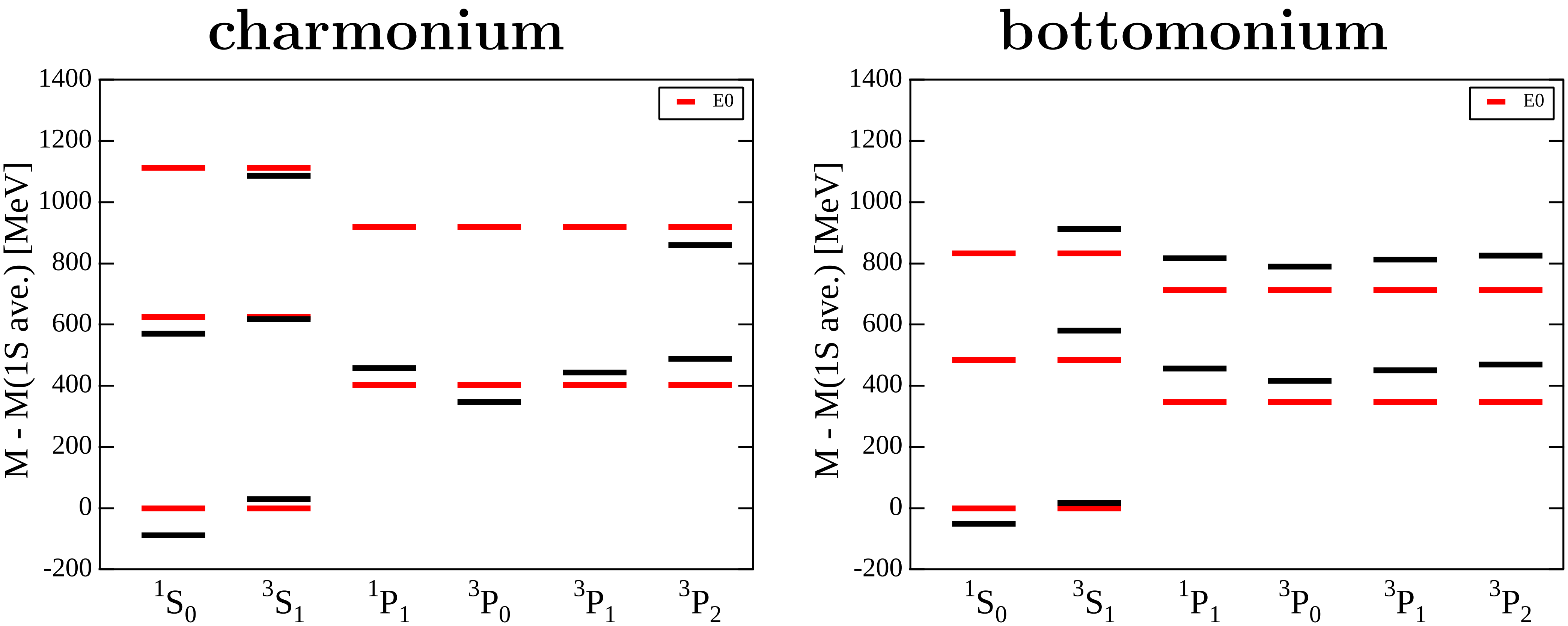
pNRQCD with lattice QCD input

- **parameters**

  - $m_b = 4.20\,\text{GeV}$, $m_c = 1.27\,\text{GeV}$
  - $V^{(0)} = -\dfrac{\alpha}{r} + \sigma r \quad (\alpha = 0.297,\ \sigma = 1.06\,\text{GeV/fm})$
  - $\delta E^{(1)} = \dfrac{2}{m} \langle \psi | -\dfrac{9\alpha^2}{16 r^2} + \sigma^{(1)} \ln r | \psi \rangle \ (\sigma^{(1)} = 0.071\,\text{GeV}^2)$
  - $\delta E^{(2)}_{ls} = \dfrac{1}{m^2} \langle \psi | \left(\dfrac{3\alpha}{2r^3} - \dfrac{(1-4\epsilon)\sigma}{2r} \right) l \cdot s | \psi \rangle \ (c_F = 1, \epsilon = 0.2)$

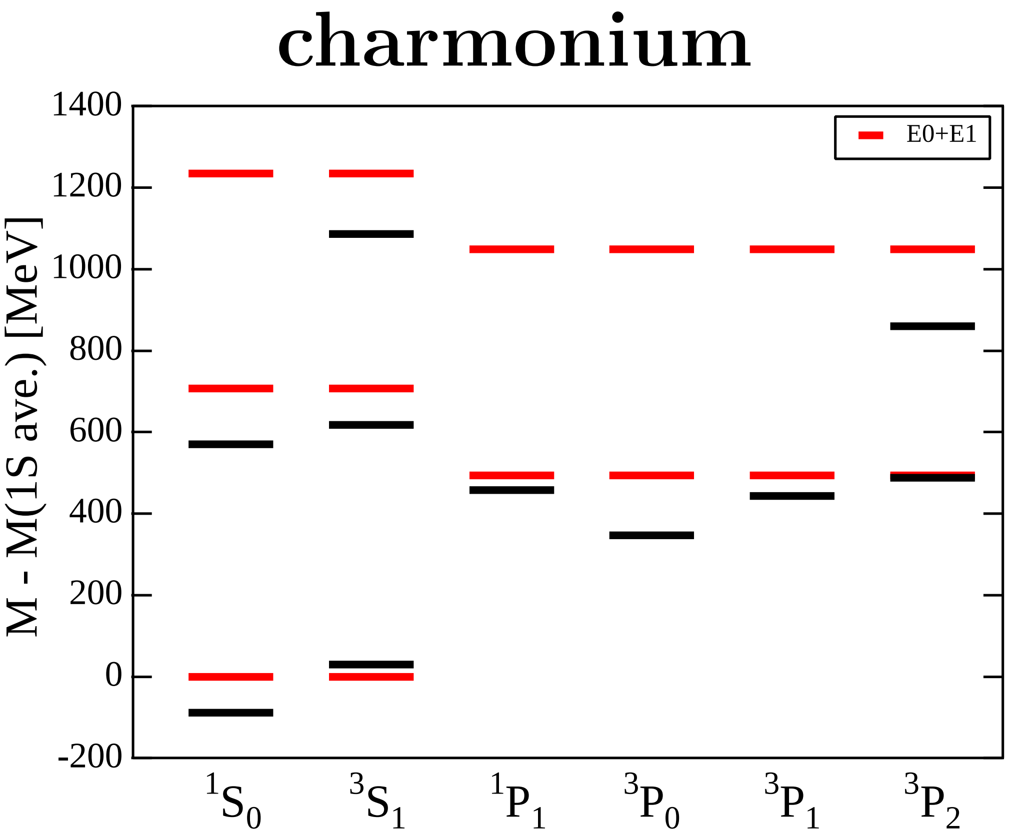
pNRQCD with lattice QCD input

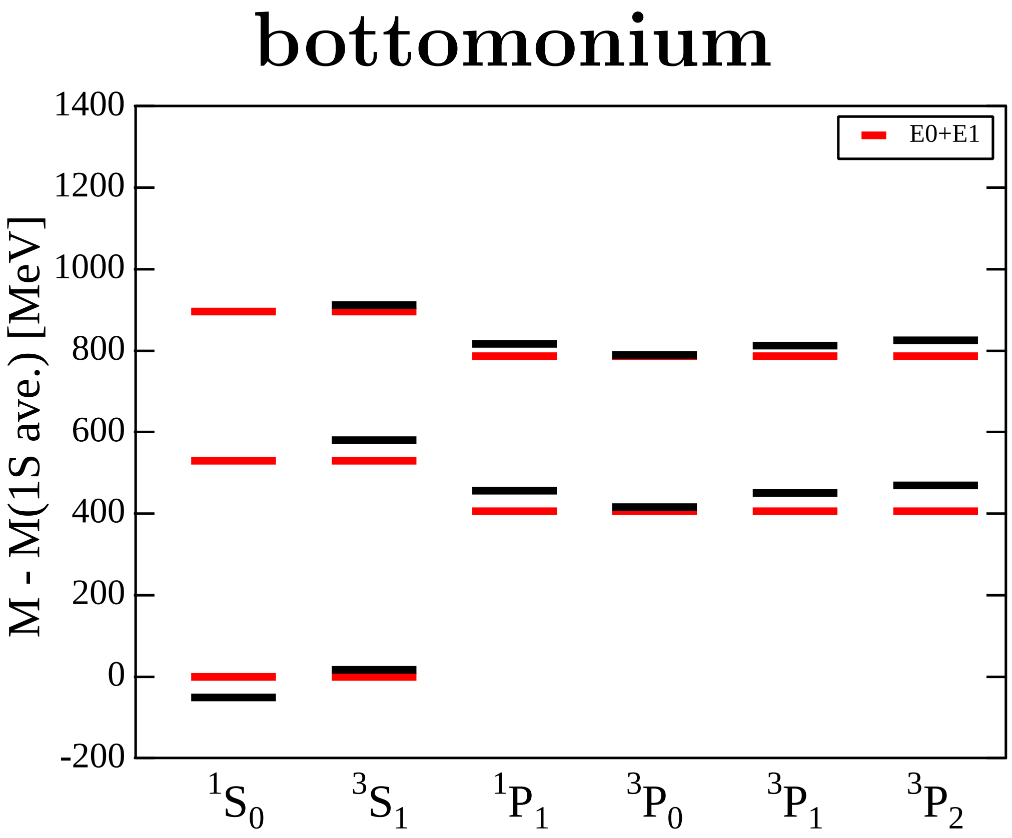
- $E = E^{(0)}$



pNRQCD with lattice QCD input

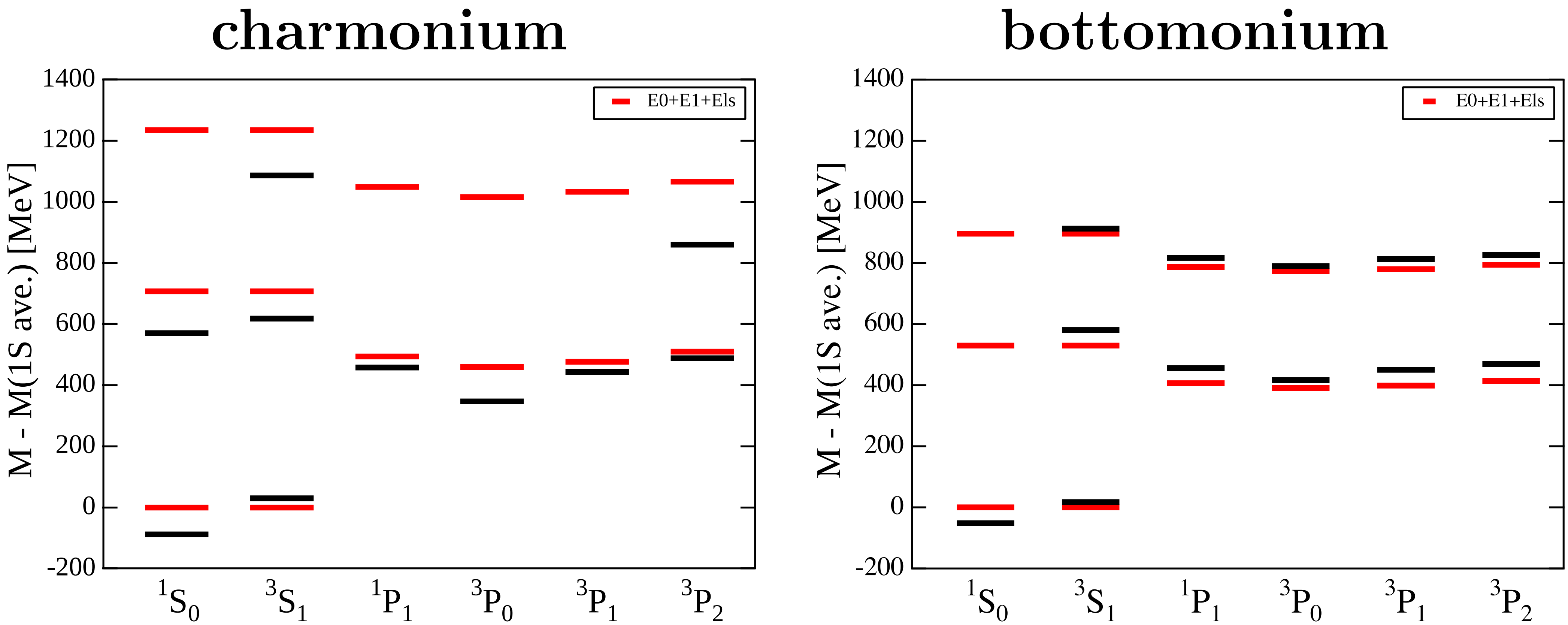
- $E = E^{(0)} + \delta E^{(1)}$





pNRQCD with lattice QCD input

- $E = E^{(0)} + \delta E^{(1)} + \delta E^{(2)}_{ls}$



Summary

- **nonrelativistic framework for quarkonium is obtained from QCD through EFTs (NRQCD and pNRQCD)**
- **nonperturbative input in pNRQCD can be computed by using lattice QCD**

  [cf. low energy constants in chiral perturbation theory]
- **quarkonium spectra are systematically studied (flavor and quantum number dependence, etc.)**

Outlook

- compute the spectrum with all $O(1/m^2)$ corrections
- take into account ...
  - short distance corrections to lattice QCD potentials
  - NLO (and further) matching coefficients in NRQCD

cf. 重いクォーク物理の進展 —QCD と量子力学系ポテンシャルの対応—

駒 佳明, 駒 美保, 日本物理学会誌 vol. 67 No. 5 (2012) 325